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THE ROLE OF SNOW SUBLIMATION IN FRONTOGENESIS

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Abstract

Recent work on the influence of moist processes at synoptic fronts has shown how condensation heating can be a strong and even a dominant factor in their development. At mesoscale lines of convection the heating is again dominant but it is thought that cooling by the evaporation of precipitation is also a crucial factor in the development and propagation. Although most simple models of synoptic fronts do not attempt to simulate diabatic cooling accurately, data from the FRONTS 87 experiment has identified evidence of a strong mesoscale downdraught below a synoptic frontal surface. There has been speculation that such a downdraught is forced by the sublimation of falling snow.

Since snow is more efficient at cooling the atmosphere, for a given precipitation rate, than evaporation of rain, rainfall parametrizations are likely to underestimate the influence of such cooling. In this paper, a suitable parametrization of diabatic cooling, based on the assumption of small stability to slantwise convection, is used to simulate a band of snow sublimation in developing fronts. The semigeostrophic equations are employed, in the study of fronts which are formed by deformation or horizontal shear of the geostrophic winds.

It is found that the cooling has little influence on the large scales of motion and does not greatly accelerate the formation of frontal singularity. However, the cross-frontal flows in the vicinity of the cooling are modified strongly, with the formation of a narrow downdraught of comparable dimensions and intensity to that suggested by the experimental data. The cooling has a weak influence on all the fields at a larger distance from the front: this is discussed in the context of the large scale, or geostrophic, response to lines of convection.

1 Introduction

Observations from the FRONTS 87 experiment (Thorpe and Clough, 1991) identified dynamical features in synoptic systems which may be interpreted as resulting from strong cooling by evaporation into descending, relatively dry air below the frontal updraught. Although vertical velocity was not measured directly, regions of strong horizontal convergence and divergence ($\frac{\partial u}{\partial x} \sim 10^{-4} s^{-1}$) were observed. It was suggested by Thorpe and Clough (1991) that a snow sublimation region of finite depth, with base at the melting level, would produce such a patch of anomalous divergence. Here, a region of cooling by the sublimation of falling snow in a developing frontal system will be modelled, using the two dimensional semigeostrophic (SG) equations, and its dynamical implications explored.

Since evaporation below the sloping updraught is also reckoned to be a fundamental component of squall line development and maintenance (as in Thorpe, Miller and Moncrieff, 1982), its role in the broader, geostrophically forced system is of immediate interest. Observations such as those of described by Meischner *et al.* (1991) show how squall lines can form in synoptic frontal regions: in contrast, the convective frontal updraught discussed by Hobbs and Persson (1982) was shown to be forced by the large scale flow. It is important to understand how the evaporative processes observed indirectly in the FRONTS 87 data interact with the process of frontogenesis.

The frontogenesis processes studied here are those of a deformation flow and of horizontal shear: these are probably the two most commonly studied frontogenetic systems. In the semigeostrophic (SG) framework the compacting of isentropes by a deformation flow is augmented by the ageostrophic wind to produce a frontal singularity on the boundary. Similarly, the temperature gradients which form as a result of horizontal shear in a baroclinic wave on the Eady basic state become singular in the SG system because of ageostrophic advection. Various studies have modelled the modification of these processes by moist diabatic effects. Analytical work such as that of Thorpe and Emanuel (1985; appendix) or Craig and Cho

(1988), tend to assume 'unconditional' heating throughout the flow. This means that diabatic cooling occurs through large regions of downward flow and is a result of the necessity to keep the analytical form of the heating simple; it is not intended to give an accurate representation of evaporative cooling. Other studies, such as those of Emanuel, Fantini and Thorpe, 1987 (EFT), or Parker and Thorpe (1994) include positive diabatic sources only—a 'conditional' parametrization—so that condensation heating is included but no evaporative cooling occurs. Indeed, Parker and Thorpe (1994) show how the diabatic heating can dominate frontogenesis in such a case and how a baroclinic wave can share properties of a squall line, even without unbalanced motion or evaporative cooling.

Few simple models have attempted specifically to simulate the role of evaporation on frontogenesis. Huang and Emanuel (1991) investigated the role of cooling by evaporation of rain beneath the updraught of a deformation front. However, as demonstrated by Clough and Franks (1991), regions of snowfall can be much more efficient at cooling a downdraught, by sublimation, than corresponding regions of evaporating rain. Since the observations of downdraughts in the FRONTS 87 data were in regions above the melting level, there is a need to represent the snow sublimation more directly. Lagouvardos *et al.* (1993) parametrized evaporative cooling in their diagnoses of ageostrophic flow from FRONTS 87 data, but again they acknowledge the need to represent the role of the snow more explicitly. In the following work, a region of cooling by snow sublimation is constructed in the two dimensional SG system, for a horizontal shear front and a deformation flow, and its role on the development is studied. More complicated (e.g. NWP) models are now attempting to incorporate ice processes explicitly: this paper provides a simpler model of the effects of sublimation at fronts, which could provide a useful comparison with NWP models.

The assumption of two dimensionality is made primarily from a dynamical standpoint; to provide a clear modification to the models involving two dimensional condensation heating. From a physical point of view also, precipitation in the stratiform region of a cold front is often observed by radar to be reasonably uniform

(e.g. Browning and Harrold, 1970). The present study is intended to expose the role of evaporative cooling on the frontogenesis process rather than its influence on the mature, developed system, which is likely to involve many more complex physical processes, such as strong frictional effects, and a wider range of scales.

In this paper, diabatic cooling by snow sublimation, i.e. conversion from snow directly to vapour, is parametrized. The general process of cooling which occurs when water converts from solid or liquid to vapour will be referred to as 'evaporation'.

2 The Semigeostrophic Equations in Two Dimensions

The semigeostrophic (SG) equations (Hoskins and Bretherton, 1972) allow the study of frontal features which are almost geostrophic in one horizontal direction but may have strong vertical component of vorticity. Observations from the FRONTS 87 experiment, analysed by Thorpe and Clough (1991) indicate that this assumption of geostrophy may be a good approximation in frontal regions. In the SG system it is significant that, in contrast to the quasigeostrophic (QG) system, the advection of momentum by the ageostrophic wind is not small and is retained. It is this property that allows the formation of fronts in a finite time.

The basic equations to be employed are precisely as used in Parker and Thorpe (1994). The SG equations are transformed to 'geostrophic coordinates', $X = x + \frac{v_g}{f}$, $Z = z$, to give analagous equations to the quasigeostrophic system. Redefining the geostrophic streamfunction by $\Phi = \phi + \frac{1}{2}(v_g^2 + u_g^2)$, the thermal wind equations may be shown to hold in the these coordinates. The semigeostrophic potential vorticity, q, is defined as,

$$q = \frac{1}{\rho}\zeta\frac{\partial\theta}{\partial Z}, \tag{1}$$

in which ζ is the vertical component of vorticity, and the potential vorticity (pv) tendency follows,

$$\frac{Dq}{DT} = \frac{\zeta}{\rho}\frac{\partial S}{\partial Z}. \tag{2}$$

where $S \equiv \frac{D\theta}{DT}$ and $T \equiv t$. Note that positive heating produces negative anomalies of pv above and positive anomalies of pv below the heating region. The inversion equation is.

$$\frac{\partial^2 \Phi}{\partial X^2} + \frac{\theta_0 f^3}{g\rho q}\frac{\partial^2 \Phi}{\partial Z^2} = f^2, \tag{3}$$

which is generally solved subject to boundary conditions of a specified field of $\theta = \frac{\theta_0}{g}\Phi_Z$.

For the two dimensional system the ageostrophic wind may be expressed in terms of a streamfunction, ψ, defined such that,

$$u_{ag} = \frac{1}{\rho}\frac{\partial \psi}{\partial z}, \; w = -\frac{1}{\rho}\frac{\partial \psi}{\partial x}, \tag{4}$$

which is calculated via the Boussinesq Sawyer-Eliassen equation:

$$\frac{\partial^2 \psi}{\partial Z^2} + \frac{\partial}{\partial X}\left(q\frac{g\rho}{f^3\theta_0}\frac{\partial \psi}{\partial X}\right) = -2Q - \frac{g\rho}{f^2\theta_0}\frac{\partial S}{\partial X}. \tag{5}$$

This is generally solved subject to the kinematic boundary condition of zero normal flow, or $\psi = 0$, on the boundaries. The geostrophic forcing term, Q, is defined as,

$$Q \equiv -\frac{g\rho}{f^2\theta_0}\left(\frac{\partial u_g}{\partial X}\frac{\partial \theta}{\partial X} + \frac{\partial v_g}{\partial X}\frac{\partial \theta}{\partial Y}\right), \tag{6}$$

The scheme for solving the SG equations consists of inverting pv using equation (3), with boundary conditions of θ, constructing Q from the geostrophic fields to solve the Sawyer-Eliassen equation and using the wind fields to advect pv and boundary θ.

3 Frontogenesis Models

3.1 Shear Frontogenesis

Shear frontogenesis is typified by the semigeostrophic dry Eady wave (Eady, 1949), for which the basic state is a geostrophically balanced baroclinic shear with height in the absence of pv gradients. As the wave develops in the troposphere, singularities of the geostrophic fields form on the horizontal boundaries; these are the surface fronts. In the semigeostrophic system the shearing of the basic state temperature gradient by the meridional winds acts to produce zonal gradients of temperature but it is the ageostrophic motion which enables singularities to form. More discussion of the dynamics of such waves and the coupling between geostrophic and ageostrophic flows is given in Parker and Thorpe (1994).

Here, using the numerical model described in Emanuel, Fantini and Thorpe (1987) waves on this basic state may be integrated into the nonlinear regime, with diabatic processes parametrized. The basic state for these waves is a linear geostrophic shear $\overline{\frac{du_g}{dZ}} = 3.0 \times 10^{-3} s^{-1}$ with a corresponding long-front temperature gradient $\overline{\frac{d\theta}{dY}}$ (constant) in thermal wind balance. Other parameter values used are tropopause height, $H = 10\ km$, Coriolis parameter, $f = 10^{-4} s^{-1}$ and static stability, $\overline{\theta_Z} = 4 \times 10^{-3} K\ m^{-1}$.

3.2 Deformation Forcing of Frontogenesis

Another process of frontogenesis, apart from the formation of singularities in baroclinic waves, comes in the consideration of a two dimensional deformation flow as a basic state. The semigeostrophic analysis of the deformation flow was investigated by Hoskins and Bretherton (1972) and, with the inclusion of slantwise condensation heating, by Thorpe and Emanuel (1985) (ThEm). The basic state winds are, $(\overline{u_g}, \overline{v_g}) = (-\alpha X,\ \alpha Y)$, with zero balanced temperature gradient. In addition, a boundary perturbation of temperature is introduced, independent of height: in ThEm this is taken as,

$$\theta'(X) = \frac{2\Delta\theta}{\pi} \tan^{-1}\left(\frac{X}{L}\right). \tag{7}$$

where $\Delta\theta$ is a fixed temperature difference, from which the inversion of pv by equation (3) will yield the long-front geostrophic wind and interior temperature field. The lengthscale, L, is determined by observing that since $w = 0$ on the boundaries, the advection by the basic state gives $L = L_0\epsilon^{-\alpha T}$, regardless of diabatic heating or cooling. Values used here are $\alpha = 10^{-5}s^{-1}$, $\Delta\theta = 12K$ and $L_0 = 5,000km$.

Since the basic state deformation flow is symmetric in the zonal direction, this form of frontogenesis can not specifically be termed a 'cold' or a 'warm' front.

4 Parametrization of Moist Processes

4.1 General Scheme

The form of diabatic heating to be used is that of ThEm in which θ is conserved except in updraughts, which conserve equivalent potential temperature, θ_e, in an atmosphere of small stability to slantwise convection (or $q >> q_e > 0$). This latter assumption is based on observations which show that θ_e can become almost constant on surfaces of constant pseudo-angular momentum (or semigeostrophic X) in frontal regions (Emanuel, 1985). The derivation was performed explicitly, for saturated flow, in EFT and leads to the expression,

$$\frac{D\theta}{Dt} \equiv S = \frac{w\rho}{\zeta}(q - \frac{\Gamma_s}{\Gamma_d}q_e), \tag{8}$$

in which ζ is the vertical component of geostrophic absolute vorticity, w is the vertical velocity, $\frac{\Gamma_s}{\Gamma_d}$ is the ratio of saturated to dry adiabatic lapse rates and $q_e = \frac{1}{\rho}\zeta\frac{\partial\theta_e}{\partial Z}$.

In practice, the frontal regions of interest involve convective motions that are not simple. The ThEm parametrization expresses these motions in terms of an overall structure, using the observations that equivalent pv is small. However, saturation is not generally observed throughout a flow of this kind. Upward motion

will rapidly reach saturation while downward flow tends to become unsaturated through adiabatic warming.

In the study of turbulence, it is often assumed that the flow consists of explicit or ensemble-averaged motion on which the turbulence is superposed. Then the influence of the turbulence may be parametrized. This is the approach to be followed here: the action of the non-resolved flow is to maintain the state of low q_e, and this is the state which the explicit flows experience. However, since the non-resolved motion involves regions which are saturated and others which are not, the ensemble average will be below saturation, as are the flows generally observed (e.g. in the FRONTS 87 data of Thorpe and Clough, 1991). Thus the assumption of saturation made in the derivation of equation (8) is only approximately satisfied. However, it will be argued here that there is good justification for using this parametrization in subsaturated ensemble flows.

The above analysis only applies when the specific humidity, h, is a thermodynamic variable $h = h(P,\theta)$; in this case, it is the saturated value of specific humidity. In order for the parametrization to be applicable to general flows, it must be assumed that the specific humidity is a function of exactly two thermodynamic variables. There is some justification for this, in that observations such as those of Thorpe and Clough (1991) show that the temperature profile often lies remarkably close to a curve of constant wet bulb potential temperature on a tephigram, even in flows which are subsaturated. In such a case, $h = h(P,\theta)$ may be a good approximation and the analysis required for (8) will hold. Thus (8) may be employed legitimately, as is generally the case, with a basic state of constant q and small q_e. However, it should be appreciated that the use of such dry and moist equivalent pv values is not consistent with a state of bulk saturation.

The diabatic heating term (8) may be taken over to the left hand side of (5) to give,

$$\frac{\partial^2\psi}{\partial Z^2} + \frac{\partial}{\partial X}\left(\tilde{q}\frac{g\rho}{f^3\theta_0}\frac{\partial\psi}{\partial X}\right) = -2Q \tag{9}$$

where $\tilde{q}$ is taken as q in unsaturated downdraughts and $\frac{\Gamma_s}{\Gamma_d}q_e$ in updraughts. Thus

the parametrization may be seen as substituting $\frac{\Gamma_s}{\Gamma_d} q_e$ for q as slantwise stability in regions of diabatic forcing. As in the dry formulation, the system retains its ellipticity provided q and q_e are both positive. the conditions for slantwise stability and conditional slantwise stability. If q or q_e were negative somewhere in the flow there would be a possibility of free solutions of (9). that is, solutions when $Q = 0$, and the uniqueness condition would be lost. However, Parker (1993) shows how this parametrization does not generally allow the pv to become negative.

4.2 Cooling Scheme

4.2.1 Basic Approach

Condensational heating may only occur in near-saturated air because the liquid or solid water develops *in situ*. Evaporative cooling may occur anywhere in a subsaturated flow because the water or ice may be imported (by precipitation) from elsewhere; the maximum rate of cooling will be that of descending air which is maintained at saturation, although other processes such as downward drag by precipitation may increase the rate of this saturation cooling by increasing the downward air speed.

Essentially one role of precipitation is to transfer heat between airmasses in a non-Lagrangian manner. For example, in a frontal region heat will be released into the atmosphere as condensation takes place in the updraught. As the precipitation falls through the air in a non-Lagrangian way, it may extract heat from the downdraught by evaporation. In the continuous condensation/evaporation system a dipole source of heat is created: the implications of such a localised dipole on the synoptic scale flow will be studied subsequently.

The work of Clough and Franks (1991) shows that to a good approximation a downdraught through which snow is falling may be taken to be saturated, at typical frontal descent rates. Clough and Franks used a one dimensional model with microphysical parametrizations of sublimation and fall speeds to show that for a given rate of precipitation, snow is much more effective at cooling the ambient

air than rain. There are two important factors in this result. Firstly, the snowflakes, for a given mass, have much higher surface areas for evaporation or sublimation than raindrops. Secondly, for a given rate of precipitation, the lower fall-speed of snowflakes, typically a factor of five less than raindrop fallspeeds, means that the density of snow in the air will be about five times greater than that of rain. Equivalently, a given snowflake will have more time during its fall for sublimation to occur. Clough and Franks were able to show that snowfall of $1 - 10\ mm\ h^{-1}$ may keep a downdraught of $10 - 30\ cms^{-1}$ saturated.

In support of Clough and Franks' results, it is commonly observed on a tephigram, such as figure 9 in Thorpe and Clough (1991), that air below the frontal surface follows a saturated adiabat above the melting layer; below the melting layer the sounding tends to be more nearly isentropic.

In the model used here, cooling is taken to occur in a prescribed snowfall region so as to maintain the downdraught saturation locally: it is this which determines that the model specifically describes snow sublimation rather than rain evaporation. The thermodynamic parametrization of ThEm for the two dimensional semigeostrophic system is employed not only in the updraught (condensation) but also in the prescribed region of saturated downdraught. The role of the melting level will be ignored for the present, as will the slow reevaporation in the rest of the broad downdraught. Browning and Harrold, 1970, estimate that around 80% of condensed water in an active frontal system precipitates out below the frontal updraught. It is assumed that the bulk of condensed water precipitates to the ground and is only available for significant levels of evaporation or sublimation in a given snowfall area.

The ThEm diabatic scheme is used under the assumption of small $\frac{q_s}{q}$, as in the updraught region. Unfortunately, in the downdraught this assumption of small stability to slantwise convection is less easy to justify physically. In the updraught it is assumed that slantwise motions, not resolved, maintain the low slantwise stability of the flow continually, but it is not clear that this may be assumed in the downdraught. In an Eady basic state, air will be advected into the downdraught at

low levels from the east, by the geostrophic shear, and it may be assumed that this air will preserve the characteristics of small slantwise stability of the updraught region. Higher up, this argument does not apply. The observations from the FRONTS 87 experiment, analysed in Thorpe and Clough (1991), for Run 3 of IOP7, indicate that low q_e may be a good assumption in the downdraught as well as the updraught, but the q_e field is somewhat noisy. The assumption of small $\frac{q_e}{q}$ will be made here for the downdraught, as a natural extension to moist up dry down (MUDD) simulations and with some justification from the FRONTS 87 data.

Other attempts to include evaporation in moist frontal systems have used a modified CISK parametrization for the diabatic cases. Thorpe and Nash (1984) and Nehrkorn (1986) describe work in which CISK heating is specified with a profile that includes cooling at low levels. Similarly, Lagouvardos *et al.* (1993) combined such a modified CISK parametrization with a condensation parametrization similar to that used here. This modified CISK method is directed at crudely modelling evaporation of rain falling directly out of a convective updraught and gives cooling which is controlled by the low level vertical velocity field. Lagouvardos *el al.* (1993) found that this form underestimated the diagnosed vertical velocity from a FRONTS 87 case study, and attribute this to a failure to account for snow sublimation accurately. By contrast, use of the ThEm scheme allows the specification of a snowfall region and is based on physically justified (via Clough and Franks, 1991) assumptions. There is local control of this elevated cooling, as it is proportional to the local descent.

Huang and Emanuel (1991) used parametrizations of rain evaporation in a deformation model of frontogenesis. This approach will be discussed in the section on deformation frontogenesis.

Although the conditions of conservation of θ_e in a saturated flow are fulfilled by the ThEm parametrization, it is important to repeat that this parametrization does not necessarily concur with bulk saturation. As in the condensation updraught, the bulk behaviour depends on the structure of dry pv. Thus the ThEm parametrization does not maximise the cooling (i.e. does not specify bulk saturation) unless

a consistently saturated state is taken. However, there is good evidence (as in Parker, 1993) that this effect is small, since the basic state values used here give a flow which is close to saturation in the mid to lower regions.

4.2.2 Prescription of Snow Region

An extreme form of diabatic parametrization is 'moist up, moist down' (MUMD), which includes both condensational heating and evaporative cooling of the ThEm form and exhibits moist diabatic processes throughout the flow. In their Appendix A, ThEm show how the assumption of small stability to slantwise convection and unconditional heating throughout a developing linear wave on the Eady basic state, leads to an analogous system to the dry Eady wave. For small perturbations the linear problem is obtained by substituting $q \longrightarrow \frac{\Gamma_s}{\Gamma_d} q_e$ in the dry system. Letting $r = \frac{\Gamma_s}{\Gamma_d} \frac{q_e}{q_0}$, the wavelength of the fastest growing mode is $\lambda_m = r^{\frac{1}{2}} \lambda_{dry}$ and the growth rate, $\sigma_m = r^{-\frac{1}{2}} \sigma_{dry}$. If, as is generally taken here, $r = 0.1$, $\sigma_m \sim 3\sigma_{dry}$, in comparison with the (numerical) moist up *dry* down result of Joly and Thorpe (1989) of $\sigma_m \sim 1.5\sigma_{dry}$. Consequently there is a factor of 2 difference between the growth rates predicted by the MUMD and MUDD cases.

Cases including diabatic effects throughout the flow, such as the above or the work of Craig and Cho (1988), are not intended to model the evaporative cooling accurately: rather the evaporation is retained in order to allow simplified analysis. In nature, precipitation removes much of the moisture in the vicinity of the front, and much of this can fall to the ground, as found by Browning and Harrold (1970). Consequently, little moisture is available for evaporation in the broad downdraught and adiabatic warming dries this region. An accurate description of these kinds of flow involves heating in the updraught and dry dynamics through most of the downdraught. However, MUMD systems do represent an extreme system for the true evaporatively cooled case, another extreme being that of MUDD. It will be important to resolve where a realistic form of evaporative cooling will lie between these two models and this is pursued here.

Given that the approximation of saturated downflow is believed to be a good

one only in regions of snowfall, the major approximation in the model is the specification of those regions of snowfall. Snow precipitates at speeds of, typically, $1ms^{-1}$, which is large compared to the magnitude of vertical motions at a front (e.g. inferred from the horizontal divergence field), but horizontal snow advection relative to the system advection may be relevent during the fall time of order one hour to reach the surface. A flake is advected horizontally with the total flow, as is the wave itself. In estimating the region in which snow sublimation occcurs, first assume that there is limitless snow and no melting level. Then the horizontal fluctuations of a trajectory from the vertical will scale as the deviation of the velocity away from the mid-level (or wave-speed) value. Near the front the ageostrophic flow tends to be opposite to the basic geostrophic shear, so taking the values for the geostrophic flow gives an estimated upper bound. For this uniform shear the upper level and lower level relative advection will be in opposite directions and tend to cancel. Thus the net horizontal distance travelled by a falling snowflake before it reaches the ground may be expected to be small. Instantaneous model relative trajectories have been plotted retrospectively for various cases and show that the deviation of the edge of the snow region (in this approximation of limitless snow) from vertical is small relative to the frontal slope and the horizontal grid-scale. The first approximation, with unlimited precipitation and no melting level, is that of a vertical cutoff between cooling region and dry downdraught, as in figure 1.

In practice the amount of snow falling at the western edge of the updraught is expected to be much less than that towards the east, because of the small depth of the cloud above, and much of the total precipitation will be converted to vapour before reaching the melting level. Browning and Harrold (1970) estimate that 25% of the precipitation in an active, albeit thundery, cold front will evaporate before reaching the surface. In this case, the amount of water and ice evaporating represents 20% of the water condensed in the updraught. The consequences of this are expected to be important since it is the strong variation in cooling with height at the edge of (below) the snow region that is thought to account for the observed patches of high horizontal divergence. To model this as simply as possible, assume

that all the snow condensed in a vertical column falls vertically and sublimes lower in the column. In this case, the integral of diabatic heating and cooling over depth must be zero:

$$\int_0^H S dz = 0, \tag{10}$$

or in the present parametrization.

$$\int_{h_c}^{h_e} \frac{w}{\zeta}\left(q - \frac{\Gamma_s}{\Gamma_d} q_e\right) dz + \int_H^{h_c} \frac{w}{\zeta}\left(q - \frac{\Gamma_s}{\Gamma_d} q_e\right) dz = 0, \tag{11}$$

where h_e is the height of the lower edge of the sublimation region and h_c is that of the condensation updraught. If $\frac{w}{\zeta}$ is estimated as being of the same magnitude in the up- and downdraughts (justified *a posteriori*) then under these assumptions a first-order estimate of the depth of the sublimation region should be taken as that of the cloud region above it, unless this intersects the melting level (see figure 1).

5 Eady Wave with Band of Snow Sublimation

5.1 Dry Mode Initialisation

With this system it is intended to simulate some of the effects of evaporative cooling in the region below the frontal updraught. First, ThEm cooling is included in a triangular region whose depth is everywhere equal to that of the heating region directly above and which extends to the ground, as in figure 1. Because of its slope with height the downdraught in the cooling region induces a band of negative $\frac{\partial w}{\partial z}$ below the band in which $\frac{\partial w}{\partial z}$ is large and positive. As a consequence strong variations in u_{ag} are expected with localised horizontal extent. Brief reference will be made to simulations in which the cutoff of cooling is vertical, below the edge of the updraught. Although it increases the integrated evaporative cooling, such a cooling region does not produce the strong pattern of $\frac{\partial u_{ag}}{\partial x}$ observed with a sloping cutoff, because $\frac{\partial w}{\partial z}$ is not so large.

The results for this system, when an initialisation with the dry normal mode is

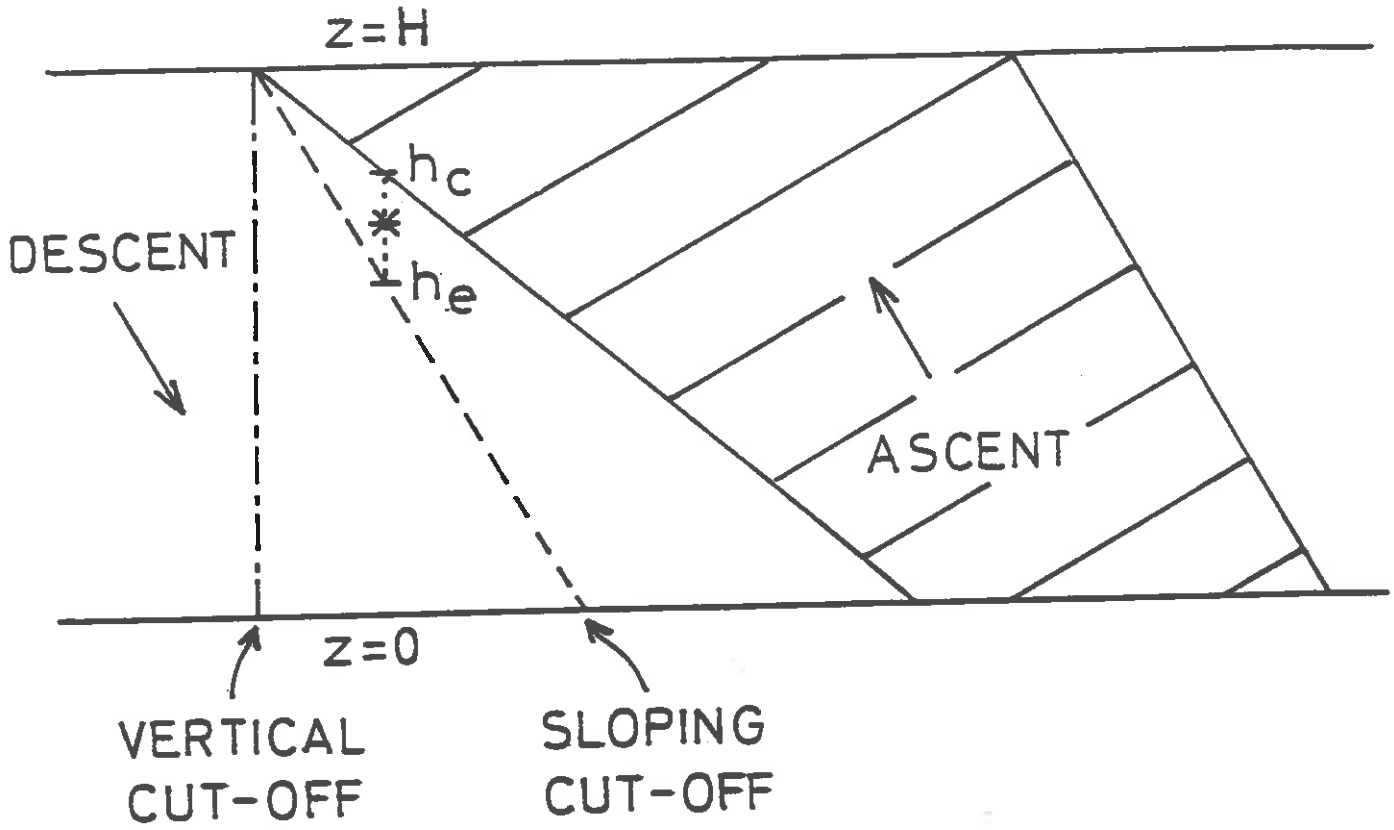

Figure 1: Schematic representation of the area in the developing wave in which cooling by sublimation of snow is taken to occur. The updraught tilts westward with height and snow falls out of this region, from height $z = h_c$. The 'vertical cut-off' assumes unlimited snow; the more realistic 'sloping cut-off' assumes that all the snow is melted by height $z = h_e$. It is assumed that the cooling region extends to the ground.

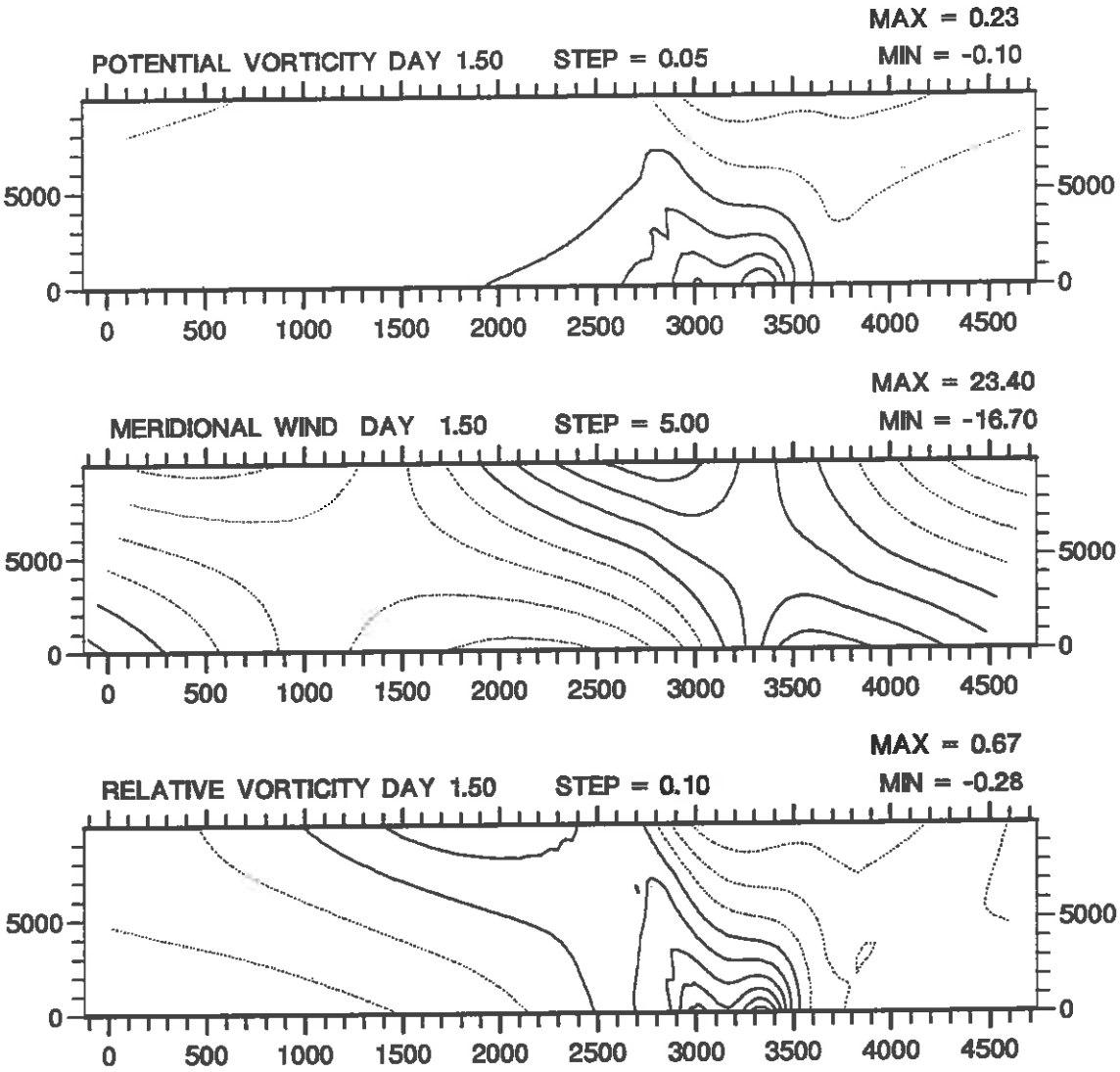

Figure 2: The geostrophic fields associated with a moist Eady wave including a cooled band of snow sublimation; day 1.5. The initial condition is the dry normal mode: ThEm heating and cooling is used, with $r = 0.1$. The wavelength is $\approx$ $4,500$ km. In this and following figures, SI units are used, except for pv (PVU). Relative vorticity is nondimensionalised with f. The contour interval is given by 'STEP' and contours with negative or zero values are dotted.

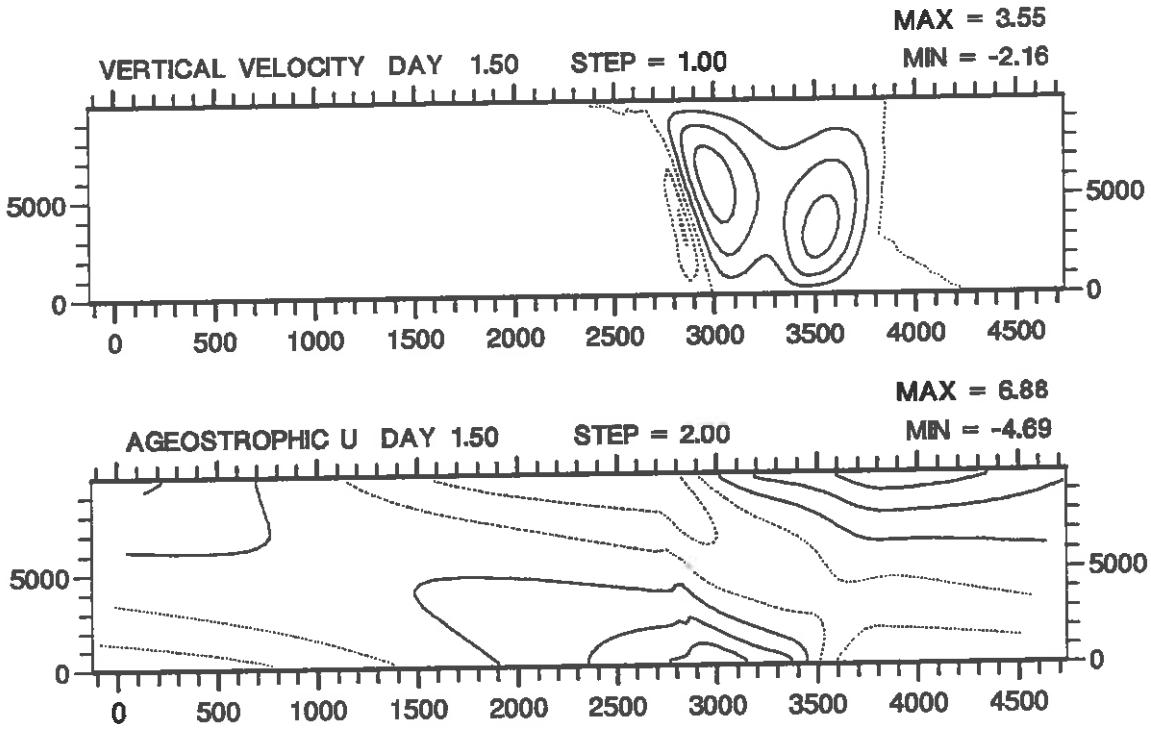

Figure 3: Ageostrophic fields for the Eady wave with cooled sublimation band; day 1.5. The strong, cooled downdraught is associated with a local anomaly in u_{ag}. similar to those observed in FRONTS 87. Units of vertical velocity are $cm\ s^{-1}$

employed, are shown in figures 2 and 3. As the wave develops, the geostrophically balanced fields seem to undergo little change from the MUDD run. The pv structure is modified slightly at low levels on the west of the updraught by the region of negative pv tendency. This causes a slight local modification of the θ and v_g fields. These effects are to cause the atmosphere to become marginally less statically stable and the horizontal shear of meridional wind to become weaker, to the west of the front (section 5.3 will consider such effects in more depth, using a vertical cutoff of cooling which increases the overall pv tendency). A slight strengthening of the westernmost vorticity maximum is evident during the development; frontal singularity is reached in about the same time as the MUDD case. Singularity occurs at the cold front, however, due to the increased ageostrophic advection locally (in the MUDD case of Joly and Thorpe (1989), the warm front reached singularity first). This effect is weak until the last stages of development.

The narrow cooling region does not couple dynamically with the cold sector in the way that the heating region is able to link with the warm sector and enhance the updraught. This is because evaporation occurs in only a small fraction of the downdraught area, at its edge, whereas the condensation heating occurs throughout the updraught. While the pv tendency in the updraught, homogeneous in (or scaling with) vertical velocity, is strongly coupled to temperature advection on the boundaries, cooling in the cutoff downdraught is not well linked to the temperature advection, even if a vertical cutoff is used. The modification of pv anomalies by the cooling has little influence on the overall wave.

Although the ageostrophic fields are strongly modified locally, their influence on advection of pv does not seem to change the balanced flows greatly, because of the natural difference of scale between geostrophic and ageostrophic advection. The feedback between geostrophic temperature distrubution and ageostrophically forced diabatic processes is strong for condensation and weak for evaporation under the present parametrization. This is expected to be a general principle since moist air will inevitably become saturated within a strong frontal updraught, whether it is convectively unstable or not, while the region of snow sublimation will never

dominate the whole downdraught of such a system, because of the restrictions to its horizontal extent. Under the unconditional (or MUMD) parametrization, the feedback between diabatic sources and the dynamical structure of the wave is identical for both updraught and downdraught. In nature, because of the precipitation of condensed moisture, this feedback is not symmetrical and the condensational heating has a dominant feedback with the dynamics, as found here.

The evaporation is, however, crucial to the determination of the ageostrophic flows and mesoscale organisation of such synoptic frontal systems. The most pronounced consequence of the inclusion of diabatic cooling is the development of the strong downdraught below the frontal updraught. The maximum of downward vertical velocity can reach $8cms^{-1}$ with a maximum in cooling, S, of $-48Kday^{-1}$, as compared with $26cms^{-1}$ and $133Kday^{-1}$ for the heating in the updraught (day 2). Associated, by continuity, is a localised peak in u_{ag} as described above. Maxima of u_{ag} are increased by $O(2ms^{-1})$ after 2 days, relative to the run with no cooling. In conjunction with the modification of the ageostrophic fields is the development of a band of horizontal divergence to the west (cold air side) of the convergence band which is to the west of the updraught (figure 4). This band of divergence is entirely due to the diabatic cooling and has the same magnitude as the convergence due to the condensation region.

The amplitude of the divergence is $O(10^{-4}s^{-1})$ by day 2. In magnitude and position this concurs with the patches of divergence observed in the FRONTS 87 data as shown in Thorpe and Clough (1991). The horizontal extent of the modelled divergence bands is around $60km$: the observed divergence bands are of comparable size. It is likely that the observed divergence pattern was associated with similar forms of precipitation due to frontal rainbands, of mesoscale cross-frontal extent and aligned almost parallel to the front, in which case three dimensional effects are weak.

The total heat release in the updraught is much greater than that removed by evaporation in the downdraught, by a factor of, typically, 10. This bears on the assumptions relating to precipitation amounts: it seems that other diabatic

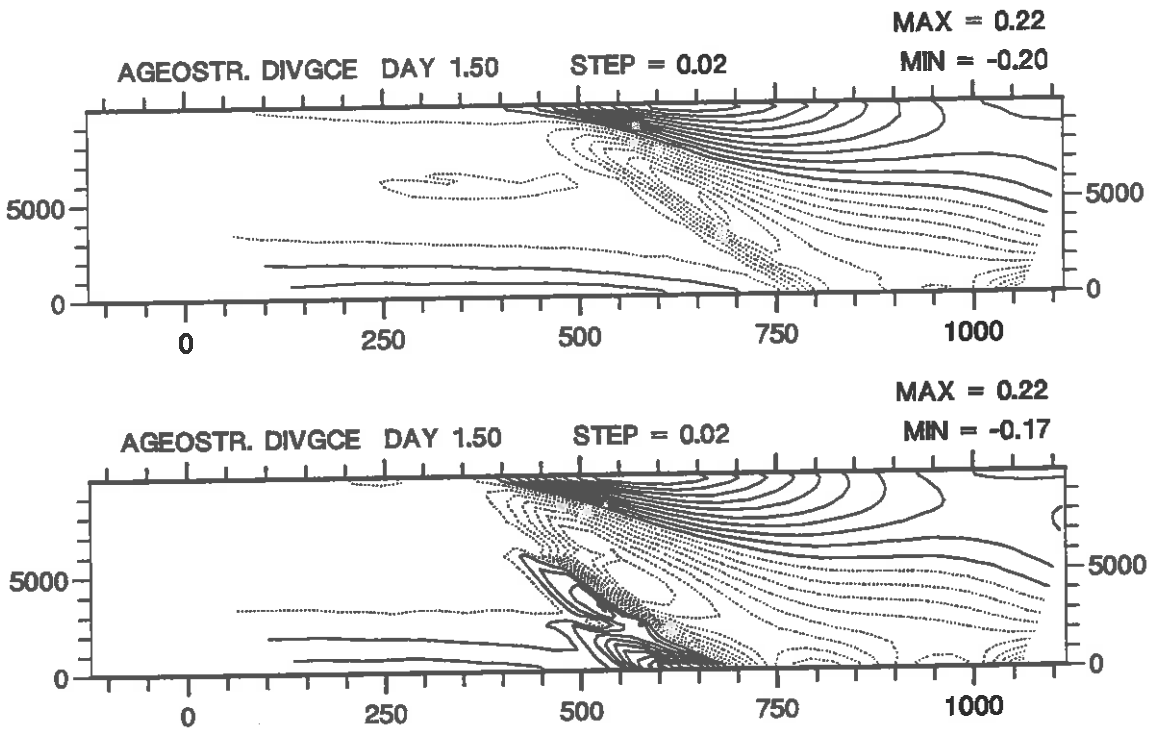

Figure 4: Comparison between divergence fields for the moist up dry down (MUDD) system (above) and the case with cooling through snow sublimation; day 1.5. The band of divergence in the cooled case lies below and to the west of the enhanced downdraught. Units of $10^{-4}s^{-1}$. The horizontal scale is in metres: $-100km \longrightarrow 1,100km$ scale.

parametrizations, such as a strong convective band, may be needed to increase the cooling and give the 25% evaporation estimated by Browning and Harrold (1970), although the 10% obtained in this model is the correct order of magnitude.

5.2 Normal Mode Analysis and Initialisation

As in Joly and Thorpe (1989), an attempt is made to obtain numerical 'normal mode' solutions to the various problems discussed under the SG formulation, using the so-called 'initial value' technique. Such solutions can be illuminating in that they highlight some of the dominant dynamics of a system and are thought to reflect the long term structure after transient behaviour has decayed. It is helpful to see these solutions, which are independent of initial perturbation, in tandem with finite amplitude simulations from a specific initial state.

The initial value method used to evaluate these normal modes is to grow a solution which remains small in amplitude so that the nonlinear terms are kept small. This is acheived by shrinking the solution to small amplitude periodically. The reduction is performed on the geostrophic streamfunction, since no pv perturbation is required for unstable modes. Solutions with modal structure in geostrophic streamfunction are sought: the initial perturbation has maximum θ' amplitude of around $10^{-6}K$. It should be noted that for the evaporatively cooled cases the approximation of strong snowfall required for Clough and Franks's result of saturated downflow will fail, although vertical velocities will similarly be reduced. The normal modes are evaluated in order to study the behaviour of the system in such a regime and compare it to the simpler canonical cases of MUMD or MUDD.

Results for the normal mode simulations with cooling are shown in figure 5, for the wavenumber of the fastest-growing solution and may be compared with the MUDD simulations of Joly and Thorpe (1989). Nondimensionally, the wavenumber of maximum growth is $k = 2.4$, calculated to $k = \pm 0.2$ and agreeing closely with the value obtained for MUDD by Joly and Thorpe (1989). There is little difference between the systems, even in the local, ageostrophic fields. The cooled downdraught below the frontal updraught is weak when compared with that com-

puted in the $1K$ finite amplitude dry initialisation. One interpretation may be found in consideration of the physical tilt of the moist updraught as well as in the lack of dynamical coupling between diabatic sources and dry wave. discussed earlier.

The unconditional (MUMD) normal mode with maximum growth corresponding to the assumed value of q_e (in these cases $\frac{q_e}{q_0} = 0.1$) has a wavelength of $\lambda_m \sim 0.3\lambda_{dry}$. This mode, with the same structure as the dry one but with a smaller horizontal scale, has a steeper updraught than the dry mode in physical space. In the limit of $q_e \to 0$ the modes would be of zero wavelength and vertical. If the moist/dry systems are regarded as a balance between a moist mode and dry mode, it can be seen that the more an updraught resembles the steep moist mode then the smaller its snowfall region beneath for sublimation. While the inclusion of this evaporative cooling may be seen as making the cross-frontal flow more like that of the moist normal mode, it is self-limiting as the moist mode exhibits a considerably narrower snow band.

In the normal mode simulations the narrowness of the snow band becomes clear, as the adjustment is complete. In the $1K$ dry initialisations there are two competing factors in the significance of evaporative cooling. Firstly the tilt of the updraught increases from a shallow-sloping dry mode with a broad snow region to a steep updraught with narrow snow band, as the wave develops. Secondly, the initial strength of the frontal downdraught is weak (that of the dry mode), giving weak evaporation. while it later becomes stronger and may yield significant cooling.

In practice the nonlinear simulations show that integrated cooling may reach 10% of the magnitude of the condensation heating by the time frontal gradients become large—approximately two days. The implication is that cooling is significant in these regions when the system evolves as a rapid development of a shallow-sloping (e.g. dry) system. The long time evolution may be self limiting, as described by the normal modes, but the nonlinear development beyond collapse to a frontal discontinuity is complicated and beyond the capacity of this inviscid SG model.

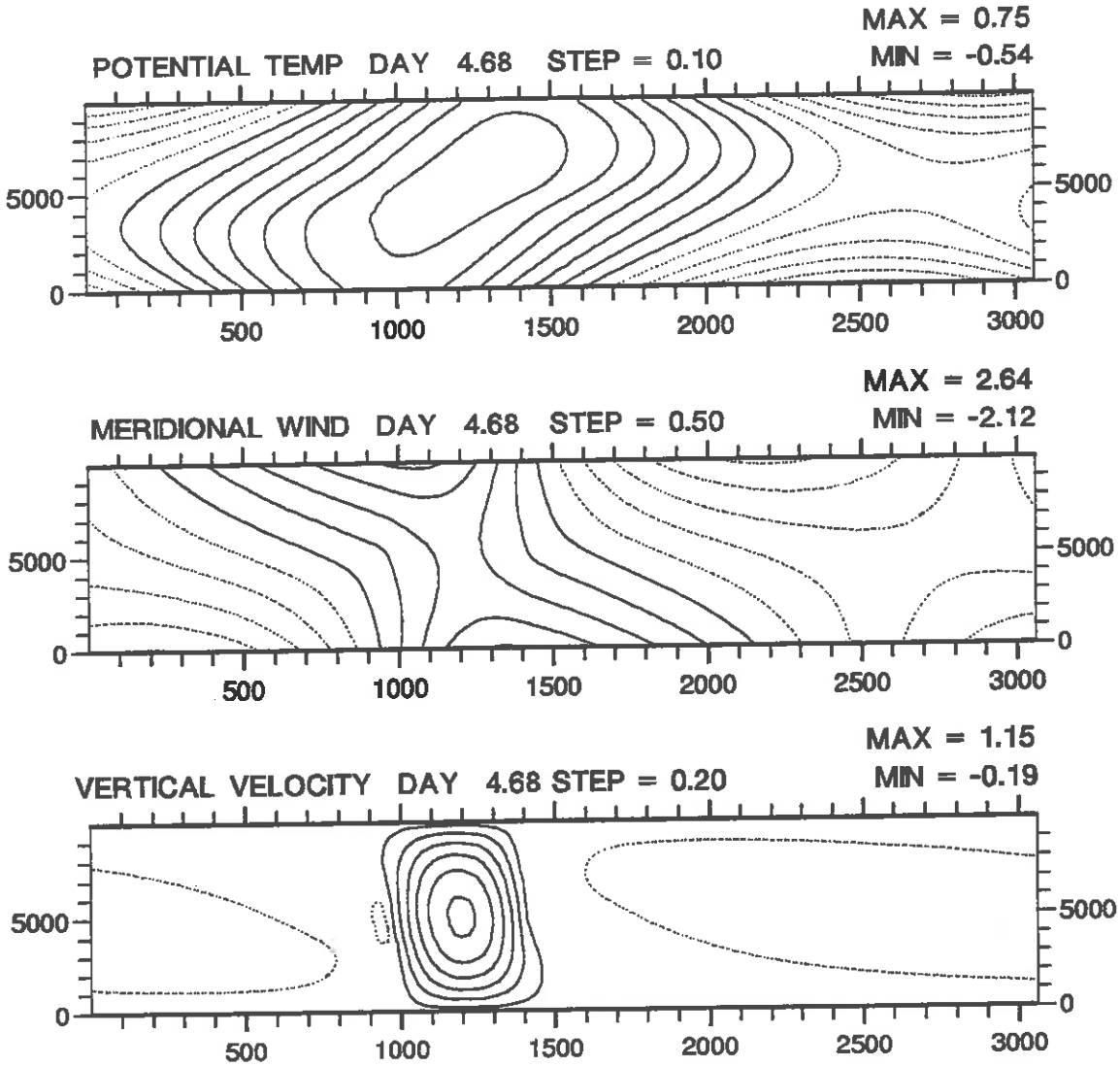

Figure 5: Fields associated with a moist Eady wave including a cooled band of snow sublimation; 'normal mode' computations. These should be compared with the equivalent structures for MUDD heating given in Joly and Thorpe (1989): they are very similar. Wavelength $\approx 3,000$ km.

Observationally, the slope of a cold front with height will generally be steeper than that of a warm front. In this way, it may be that evaporative processes are more effective beneath a warm frontal than a cold frontal updraught. In the waves described here, however, the updraught region tilts westward with height and the snow band parametrization is not suitable for consideration of warm frontal modification. The deformation front, which is symmetrical in frontogenetic forcing, is more apt for discussion of warm front modification by snow sublimation and this is pursued in section 6.

5.3 Sensitivity of Geostrophic Flow to the Cooling

When the nonlinear routine is initialised with a normal mode for the problem, a more direct comparison with the MUDD case may be made, since the normal modes of each are similar. In this case the cooling region will be taken to extend vertically below the updraught, with a vertical cutoff, to maximise the cooling under the assumptions of this model.

Although the effect of the cooling on the geostrophic fields is weak, its nature is interesting. Figure 6 shows the influence of the diabatic cooling on the meridional wind. Both cases are close to collapse of the model to a vorticity singularity at the boundary. To the west of the updraught, positive pv anomalies generated by the diabatic heating of the updraught are advected westward by the basic shear. When evaporative cooling occurs this serves as a negative source of pv on the positive anomaly as it advects through the downdraught. The low level pv to the west of the updraught is reduced in the case with cooling. This is manifested most directly in the fields of meridional wind and potential temperature.

The influence of the pv modification on the local meridional wind has an important physical implication for the flow locally. Because the pv anomaly to the west of the updraught is reduced in the cooled case, the shear in this region is small (in addition, this region is close to the zero-line of potential temperature, so the shear induced by boundary θ perturbations is small). Thus the negative meridional jet has a broad maximum, extending to the region of high shear. There

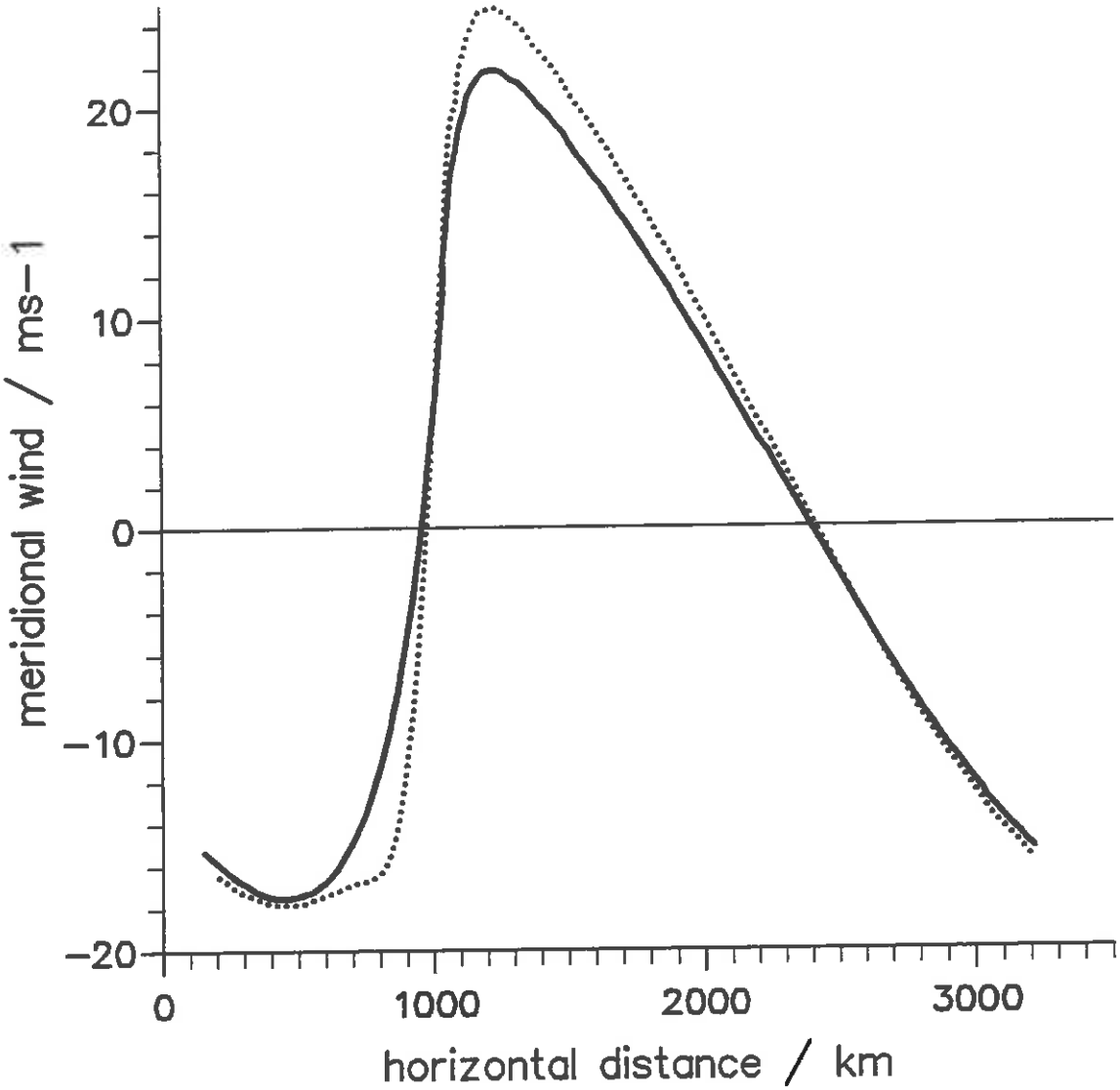

Figure 6: The meridional velocity fields at the lowest level for the MUDD (solid line) and vertical snow band (dotted line) cases. The reduced shear to the west of the front when diabatic cooling occurs is due to the reduced pv anomaly. It can be seen how this leads to stronger northerly flow behind the front, giving a positive feedback between diabatic cooling and cold advection.

is increased negative thermal advection to the west of the vorticity front when cooling is included: the effect of the diabatic cooling is to increase the meridional cold advection, and as such produces a positive feedback.

This positive feedback between diabatic cooling and meridional advective cooling is manifested in the solution of the Sawyer-Eliassen equation. Using the rule-of-thumb that $w \propto v_g$ from the Q-forcing of the S-E equation, as derived in Parker and Thorpe (1994), the geostrophic forcing of the downdraught, to the west of the vorticity front, is stronger when the cooling occurs. By reducing the pv (and shear) to the west of the front, the cooled region has increased its favourable geostrophic forcing locally.

It remains true that these effects only seem to be significant locally (and are amplified by the rather artificial assumption of a vertical cutoff to the cooling), but in the vicinity of the front, there seems to be a positive feedback between the diabatic cooling and its forcing through cold advection. In these simulations, initialised with the normal mode, the initial pv structure will be more favourable for a strong downdraught, in contrast to runs initialised with the dry mode in which some adjustment will need to occur.

In addition, the increase in cold advection behind the cold front is a factor which will bear on the intensity of the front: Fovell (1991) notes how such increased advection in a squall line model can enhance the longevity of a system. However, there is no evidence of a pool of cold air forming in the cooled system. The balanced model would not be expected to describe such a phenomenon well, because of the inherently gravity-dominated nature of a mesofront. Furthermore, since the motion is assumed to occur at stability to slantwise convection, the cooling does not overcome adiabatic warming in the descent region.

6 Deformation Front with Sublimation Cooling

This section extends the work of ThEm by including a parametrization of cooling by sublimation of snow below the frontal updraught, using the same cooling

assumptions as employed above in the Eady wave simulations. A similar case was studied by Huang and Emanuel (1991), who included the parametrization of evaporation by rain, storing the water vapour mixing ratio and liquid rainwater as explicit variables in the problem and using a specific empirical relationship to evaluate the evaporation. Clough and Franks (1991) show how the evaporation of rain is relatively inefficient at cooling the atmosphere and consequently Huang and Emanuel's (1991) work is likely to underestimate the effects of snow sublimation. Also, as discussed earlier, the evaluation of relative humidity under the heating parametrization of ThEm requires either an interpretation of the updraught as some kind of ensemble, as in CISK, or a specific relationship between dry and moist equivalent pv. Huang and Emanuel assume relative humidity of 100% in the updraught, with constant dry pv and very small equivalent pv: these assumptions are not physically consistent, although Parker (1993) suggests that the inaccuracy is likely to be small in the lower regions of the atmosphere. The present system differs in that it uses simpler parametrizations which are based on the work of Clough and Franks (1991) and, although it likewise fails to exhibit bulk saturated flow, closer modelling of such processes would probably need explicit parametrizations of cloud ice. As such, although this study involves various unavoidable approximations, it may provide a benchmark for more explicit modelling.

The solution for MUMD (or 'unconditional') heating was given in Castelli, Bras and Emanuel (1993), and an approximate analytical solution of the conditional case was presented. The results indicated that for small q_e, the MUDD form reaches singularity much faster than MUMD. They state that this is related to the differential ageostrophic advection across the front.

Figures 7 and 8 compare the fields of pv and vertical velocity, respectively, for MUDD and sublimation cooled deformation fronts, after one day of development. The band of cooling has a sloping cutoff, as before, so the snow band has equal depth to that of the cloud above. The noise in the pv field is due to the fact that the cutoff of diabatic cooling crosses X-surfaces in the numerical grid. This is not expected to be important, as the inversion of pv acts as a 'smoothing' operator, as

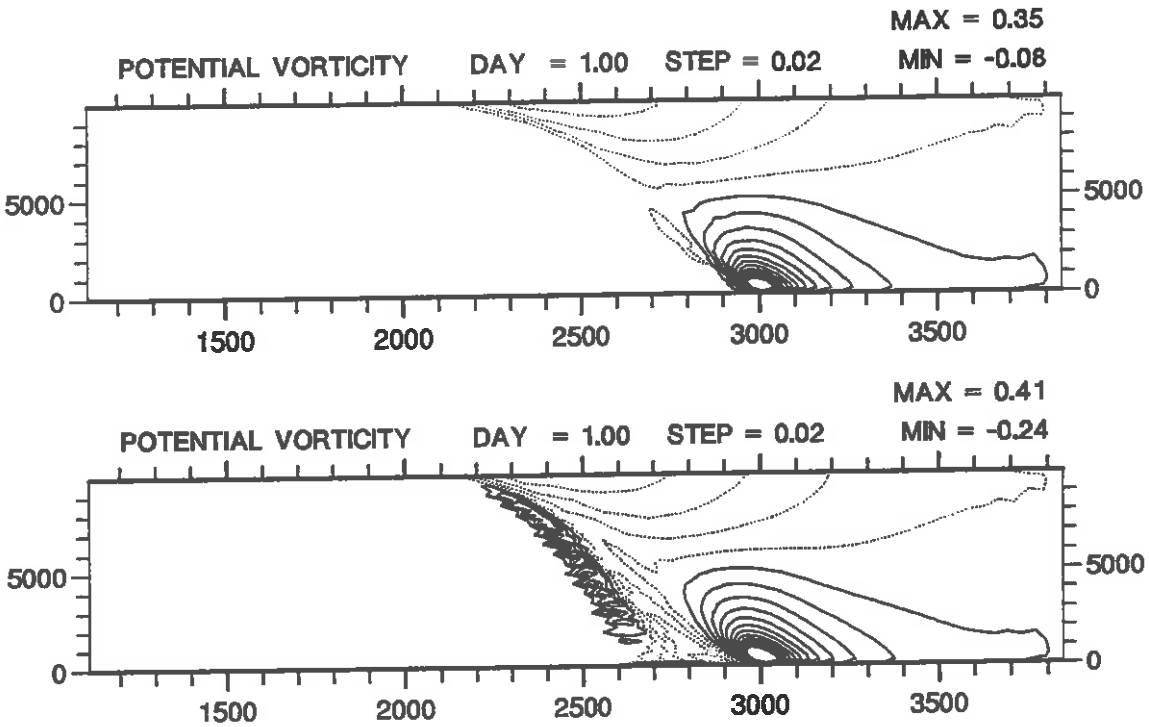

Figure 7: The fields of potential vorticity for a moist up, dry down (MUDD) deformation front and the same case with inclusion of sublimation cooling. The 'noise' in the cooled case is related to the snow cut-off crossing momentum surfaces in the numerical grid, but is not thought to invalidate the results, as the inversion of pv is a 'smoothing' operation. Horizontal extent of domain 2,500 km.

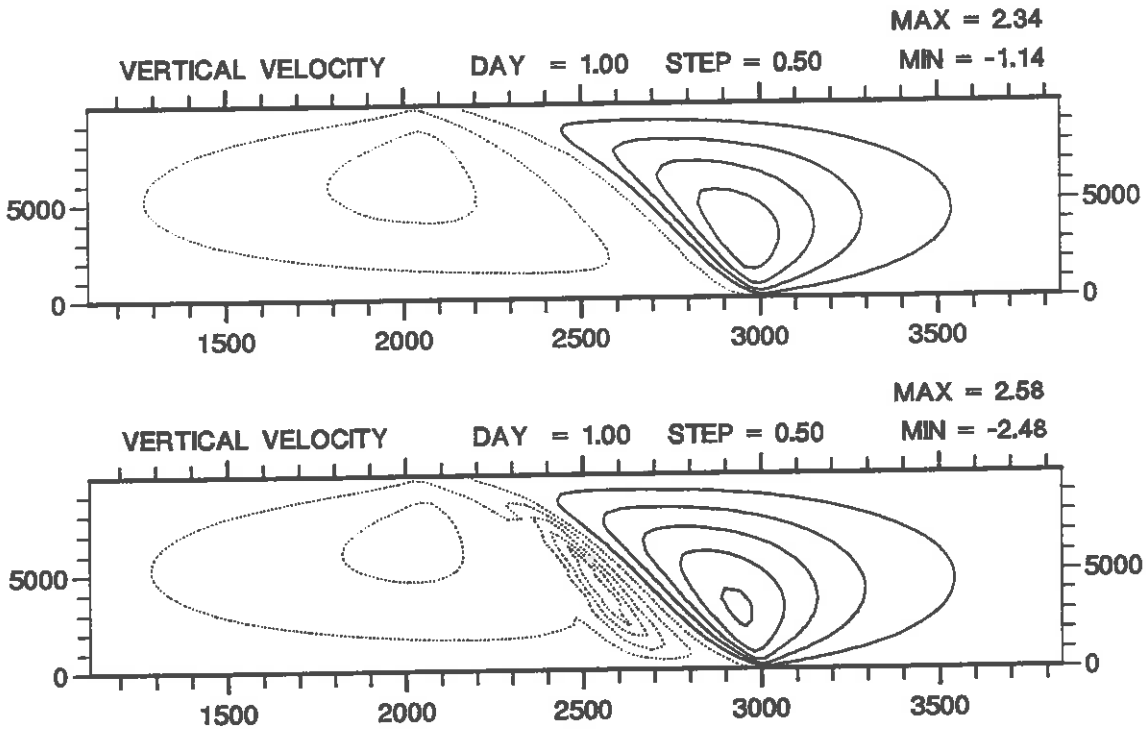

Figure 8: The fields of vertical velocity for a MUDD deformation front and the same case with inclusion of sublimation cooling. Horizontal extent of domain $2,500$ km.

will be discussed subsequently.

The cooled case shows a markedly stronger downdraught than both the MUDD model and the rain evaporation model of Huang and Emanuel (1991). The downdraught reaches a magnitude comparable with that of the updraught, in contrast to the Eady simulations. The more intense downdraught compared to the work of Huang and Emanuel (1991) is due to the fact that the relatively efficient cooling of the air by snow rather than rain has been modelled here. The ageostrophic divergence pattern associated with this strong downdraught is shown in figure 9. The band of divergence below the sloping downdraught reaches a magnitude of order $10^{-5}s^{-1}$, compared with typical values of $10^{-4}s^{-1}$ for the Eady basic state.

The pv structure also shows marked differences from the MUDD form, with the cooling producing a negative anomaly at low levels and positive aloft. In the Eady wave the upper positive anomaly is advected by the basic state and swamped by the strong effects of the condensation heating while at low levels the negative tendency due to cooling acts on an already strong positive anomaly. In this deformation flow,

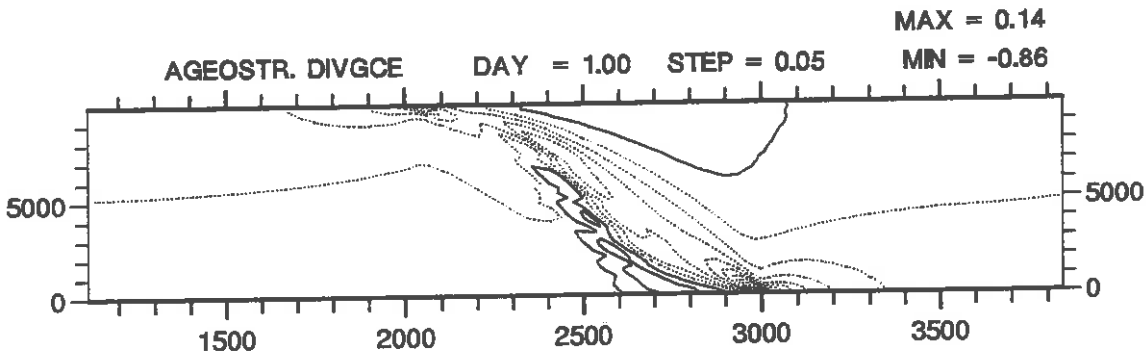

Figure 9: The ageostrophic divergence field of the deformation front with a band of sublimation cooling. As in the Eady wave simulations, the band of divergence observed in FRONTS 87 has been qualitatively reproduced. The axis scales are in metres; the horizontal scale has been altered to emphasise the slope of the system. Units are $10^{-4}s^{-1}$. Horizontal domain $1,200 - 3,700\ km$.

close to the surface of $w = 0$ the cross-frontal geostrophic wind is weak and the pv anomalies are not strongly advected. The low level negative anomaly is relatively large (-0.24 PVU compared with the basic state pv of .33 PVU). There is also an upper level positive pv anomaly where momentum surfaces cross the cooling band. Although it is relatively weak this upper level positive anomaly may be seen to represent an equivalent change in the height of the tropopause in the vicinity of such a front (generally the stratosphere has pv in excess of 2 pv units). Despite the modified pv structure, the geostrophic fields such as the meridional wind do not change greatly, as in the Eady simulations.

In the deformation front, the tilt of the updraught is determined largely by the boundary structure of potential temperature and this is dominated by the basic state advection due to the deformation field. The modifications to the boundary advection come only from the u_{ag} field, when diabatic processes are involved. Consequently the tilt does not change significantly between these runs and the amount of cooling is not influenced by this effect. This is one way in which evaporation acts more effectively in the deformation front than in the Eady wave. Unlike the Eady case, the cooling does not act to increase the updraught steepness, a process which was seen to be a negative feedback on the cooling.

As has been stated earlier, the ThEm parametrization, as used, with constant q and small q_e, probably gives an unrealistic distribution of heating with height. With a more suitable initial profile of q, the heating would be more concentrated at low levels and may intensify the downdraught locally. Huang and Emanuel's (1991) downward velocity maximum produces a band of u_{ag} maximum at low levels, giving divergence which has a local magnitude of around $10^{-5}s^{-1}$.

Huang and Emanuel (1991) found that the intensification of the downdraught and the associated modification of u_{ag} acts to increase the frontogenesis, typically by around 10%. The results using the ThEm form for cooling confirm the acceleration of frontogenesis but by larger amounts; typically by about 20% for a sloping cutoff to the cooling and 50% with the vertical cutoff. Recall that the modified, analytical form of this problem, as studied by Castelli, Bras and Emanuel (1993), showed growth rate to be much more rapid, for small q_e, in the conditionally heated case than the unconditional case. Again, as in the Eady modes, the system with a band of cooling seems to resemble the MUDD form more closely than the MUMD.

7 Far Field Response to Diabatic Sources

Eliassen (1959) explored the general role of source terms on the S-E equation in forcing of vertical motion. He found that within a uniform diabatic source region the flow follows momentum surfaces, while outside it decays in a similar way to the flow due to a point source of forcing.

In the Sawyer-Eliassen equation, the streamfunction at a large horizontal distance from a front may be regarded as the far-field response to localised heating in the vicinity of the front. Parker (1993) discusses this more explicitly. Mathematically, the dipole component of the diabatic source has only a weak influence on the far field. The monopole component will be dominated by the heating, which is considerably stronger than the cooling. In the consideration of cooling below a frontal updraught, this means that, as observed in the numerical simulations, the details of the two dimensional heating in the region of the front do not affect the

far field. The influence of the diabatically forced region on the flow a long way from the front is hardly altered by the inclusion of cooling from the case in which the heating was only positive.

Since the first order effect of the diabatic structure is represented by the monopole integral of the heating, $\int S dx dz$, the mesoscale dynamics of the updraught region need not be represented, to obtain an understanding of how moist processes affect the large scale two dimensional development. Global parametrizations such as CISK and the ThEm scheme, which represent an ensemble of mesoscale instabilities, can be used in the zone at which such instabilities occur. Accurate representation of the diabatic processes depends, from the point of view of the synoptic system, far more on the judicious application of these parametrizations than on the mesoscale structure of heating.

8 Summary and Overview

Unlike the updraught condensation processes, cooling by snow sublimation plays only a minor role in the synoptic development of a baroclinic wave. Its very nature, coupled only indirectly to the downdraught by the geometry of the heated updraught, means that a region of cooling has little effect on the large scale structure of the flow as the front forms. As a corollary, it seems that such diabatic processes, if sufficiently decoupled from the overall frontal development, may be well modelled by taking the large scale fields as a basic state.

The significance of cooling processes comes on consideration of smaller scales of motion and velocity. Bands of snow sublimation may produce marked frontal downdraughts and associated horizontal flows on the mesoscale, provided the tilt of the system is sufficient. However, in the case of shear frontogenesis, the development of such downdraughts may be seen as self limiting in their horizontal extent and appears to be strongly controlled by the other dynamics. A weak but noticeable effect on the geostrophic flow is to increase the cold advection by the meridional wind in the vicinity of the downdraught: this is consistent with a positive feedback

between the cooled ageostrophic flow and its geostrophic forcing. However, the acceleration of frontogenesis is weak, especially in the case of horizontal shear forcing. There is no evidence in these simulations of a 'cold pool' forming at the front, as has been suggested by observations such as those of Hobbs and Persson (1982). This may be because the model is run with stability to slantwise convection and includes neither friction nor the influence of gravity waves.

The results of this paper describe the influence of evaporative cooling on balanced systems. This is in part motivated by the observations from FRONTS87 described above, and in part to link the dynamics with smaller scale features. Squall lines are not generally seen as balanced systems, because of their relatively short timescales and lengthscales, but they share certain dynamical features with active synoptic fronts, as discussed by Parker and Thorpe (1994). Fovell (1991) described how an along-line temperature gradient could favourably feed back into the squall line cold pool, through the meridional wind, as in shear-forced balanced models. Similarly, the effects noted in this work for balanced flows will have some influence on a squall line. For instance, it has been shown that the cooling acts to reduce the low level pv anomaly to the west of a front. This process in a squall line would have a similar effect, to reduce the shear of meridional wind behind the line and to increase the magnitude of this equatorward wind close to the updraught, again increasing the advective cooling locally. From the opposite perspective, it is seen that the cooling, which is thought to be essential to the maintenance and propagation of a squall line, has little influence on the overall balanced synoptic system.

The balanced dynamical model would not be expected to reproduce squall line dynamics, so the influence of the cooling in the genesis of fast-moving rainbands in frontal zones is not explored. However, the results suggest that the large scale or balanced response to a squall line, which may be crucial to the preconditioning of the inflow to the line, is determined principally as a response to the monopole source of the heating, dominated by condensation. In addition, the fact that the cooling is of secondary importance in the balanced flows gives credence to the idea

that 'conditionally' heated waves which do not involve evaporative cooling, such as that of Parker and Thorpe (1994), may be able to reflect aspects of squall line dynamics.

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